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Sem - VI
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# HEISENBERG'S UNCERTAINTY PRINCIPLE

The de-Broglie wavelength associated with a moving particle travelling with uniform velocity v is given by

$$\lambda = \frac{h}{mv}$$

This is a monochromatic wave of infinite extent. The phase velocity $V_p$ of such a monochromous de-Broglie is given by

$$V_p = \nu\lambda = \frac{h\nu}{h/\lambda} = \frac{E}{P} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

and is thus always greater than the velocity of light in vaccum. It is, therefore, theoretically impossible for monochromatic de-Broglie wave train to transport a particle or carry energy. The phase velocity merely represents the rate at which a given phase of a mono-chromatic wave train advances and a purely a mathematical concept.

Further the stability of a material particle like an electron demands that it should be concentrated over a small region of space at any given instant of time. Thus the mass and charge of a particle like the electron are localised entities whereas the de-Broglie wave with which we represent the moving particle is of infinite extent by virtue of its being monochromatic.

By taking a group of waves of nearly equal wavelength centered round the de-Broglie wavelength $\lambda = \frac{h}{P}$, we obtain a wave-packet the amplitude of which is different from zero only in a certain small region of space having dimension of the particle.

This small region of space is associated with the position of the particle and this wave packet travels with the velocity known as group velocity, which is equal to the particle velocity $v$.

Hence, instead of associating a single mono-chromatic de-Broglie wave with particle $v$ associate with it a wave-packet consisting of a group of wave of nearly equal amplitude centered round de-Broglie wavelength $\lambda = \frac{h}{p}$ of the particle. The wave packet has the dimension of the localised particle and travels with the same velocity as the particle.

The association of a group of waves with a moving particle means that the position of the particle at any instant of time cannot specified with any desired degree of accuracy. All that we can say is that the particle is somewhere within the group of waves ~~example~~ i.e. within a small region of a space. Thus a certain inaccuracy will always creep in ascertaining the position of a moving particle like the electron, proton etc. With the similar reasoning it can be argued that a certain amount of inaccuracy will creep determining the velocity and hence the momentum of the particle.

Heisenberg's uncertainity principle states that in any simultaneous determination of the position and momentum of a particle, the product of the uncertainities is equal to or greater than planck constant $h$.

i.e. $\Delta x . \Delta p_x \geq h$

for motion along X-axis, where $\Delta x$ is the fundamental error or uncertainity in the measurment of the position and $\Delta p_x$ the fundamental error or uncertainity in the measurment of momentum (velocity)

A more accurate calculation gives

$$\Delta p_x \Delta x \geq \frac{1}{2} \frac{h}{2\pi} = \frac{1}{2} h$$

According to classical ideas, it is possible for a particle to occupy a fixed position and have a definite momentum and we can predicte exactly its position and momentum at any time later. According to uncertainity principle, it is not possible to determine accurately the simultaneous values of position and momentum of a particle at any time. If it is desired to reduce the error, in the determining of position i.e to reduce $\Delta x$, then this can only be done at the expense of accuracy in determining the momentum and error involved in $\Delta p$ will increase.

It should be born in mind that uncertainities are not due to imperfections of the measuring instruments or way of measurments. These are inherent due to wave nature of the particle.